If we now substitute $a+r$ for b and $2r$ for r in (12) we shall find for the sum of series (2)

$$S_x^{(n)} = \frac{(2r)^n}{n+1}x^{n+1} + (2r)^{n-1}ax^n + \frac{n(2r)^{n-2}}{6}(3a^2-r^2)x^{n-1}$$
$$+\frac{n(n-1)(2r)^{n-3}a}{6}(a^2-r^2)x^{n-2}+\dots \qquad (14)$$

If we make $a=0$ and $r=1$, we shall have

$$1^n+2^n+3^n+\dots+x^n=\frac{1}{n+1}x^{n+1}+\frac{1}{2}x^n+\frac{n}{12}x^{n-1}+\dots \qquad (15)$$

$$1^n+3^n+5^n+\dots+(2x-1)^n=\frac{2^n}{n+1}x^{n+1}-\frac{n.2^{n-3}}{3}x^{n-1}+\dots \qquad (16)$$

If we make $a=0$ and $r=2$ in series (1) we shall have

$$2^n+4^n+6^n+\dots+(2x)^n=\frac{2^n}{n+1}x^{n+1}+2^{n-1}x^n+\frac{n.2^{n-2}}{3}x^{n-1}+\dots$$

REVISED SOLUTION OF PROBLEM 218.

EDITOR ANALYST:

MR. MEECH, the ingeneous proposer of problem 218, having furnished me with the data from which that question was constructed, and requested me to make a general solution, under fuller conditions, for publication in the ANALYST, I hereby cheerfully comply with his request.

GEORGE EASTWOOD.

The problem, under its new aspect, may be stated as follows:

Required the separate rates of dividend of two insolvent estates connected as follows:

JOHN DOE'S ESTATE. Direct liabilities $=\lambda$; his endorsements for Richard Roe$=\lambda_1$ *less* a first dividend on the same to be paid out of Roe's estate. His net assets $=a$ to be increased by dividend on account, $=\beta'$, due from Roe's Estate.

RICHARD ROE'S ESTATE. Direct liabilities $=\lambda'$; his endorsements for John Doe$=\lambda_2$, less a first dividend on same to be paid out of Doe's estate. His net assets$=a'$ to be increased by dividend on account, $=\beta$, due from Doe's estate.

We have Doe's direct liabilities $=\lambda$; his endorsements $=\lambda_1$;
Roe's direct liabilities $=\lambda'$; his endorsements $=\lambda_2$;
Doe's net assets $=a$;
Roe's " " $=a'$;
Doe's account due from Roe's estate $=\beta'$;
Roe's " " " Doe's " $=\beta$.

Therefore, x being the rate per cent of dividend on Doe's estate and y the rate on Roe's, we have

First dividend on Roe's endorsement for Doe $= \lambda_2 . \frac{1}{100} x$;

First dividend on Doe's endorsement for Roe $= \lambda_1 . \frac{1}{100} y$.

$$\lambda_2 - \lambda_2 \frac{x}{100} = \lambda_2 \left(\frac{100-x}{100}\right) = \text{balance of Roe's endorsements,}$$

$$\lambda_1 - \lambda_1 \frac{y}{100} = \lambda_1 \left(\frac{100-y}{100}\right) = \text{balance of Doe's endorsements.}$$

$\alpha' + \frac{1}{100}\beta x =$ amount to be dividend among Roe's creditors;
$\alpha + \frac{1}{100}\beta' y =$ " " " " " Doe's " ; also

$$\left[\lambda' + \lambda_2 \left(\frac{100-x}{100}\right)\right] \frac{y}{100} = \text{final dividend on Roe's state,}$$

$$\left[\lambda + \lambda_1 \left(\frac{100-y}{100}\right)\right] \frac{x}{100} = \text{" " " Doe's estate.}$$

Hence, from the nature of the question,

$$\left[\lambda' + \lambda_2 \left(\frac{100-x}{100}\right)\right] \frac{y}{100} = \alpha' + \frac{\beta x}{100}, \tag{1}$$

$$\left[\lambda + \lambda_1 \left(\frac{100-y}{100}\right)\right] \frac{x}{100} = \alpha + \frac{\beta' y}{100}. \tag{2}$$

From (1) and (2) we deduce, respectively,

$$y = \frac{100(100\alpha' + \beta x)}{100\lambda' + \lambda_2(100-x)} \text{ and } y = \frac{100[(\lambda + \lambda_1)x - 100\alpha]}{\lambda_1 x + 100\beta'}.$$

Equating these values of y, reducing and arranging like powers of x, we find

$$x^2 - \frac{100[(\lambda+\lambda_1)(\lambda'+\lambda_2)+\alpha\lambda_2-\alpha'\lambda_1-\beta\beta']}{\beta\lambda_1+\lambda_2(\lambda+\lambda_1)} x = -\frac{(100)^2[\alpha(\lambda'+\lambda_2)+\alpha'\beta']}{\beta\lambda_1+\lambda_2(\lambda+\lambda_1)}. \tag{3}$$

Put the coefficient of x in (3) $= 100A$ and the right-hand member of the equation $= (100)^2 B$, then equation (3) becomes

$$x^2 - 100Ax = -(100)^2 B,$$

whence $$x = 50A \mp 50\sqrt{(A^2 - 4B)},$$

the upper sign of which is to be applied, and thence the value of y is easily determined.

If, as in the former problem, $\beta' = 0$, then the final equation in x becomes

$$x^2 - \frac{100[(\lambda+\lambda_1)(\lambda'+\lambda_2)+\alpha\lambda_2-\alpha'\lambda_1]}{\beta\lambda_1+\lambda_2(\lambda+\lambda_1)} x = -\frac{(100)^2\alpha(\lambda'+\lambda_2)}{\beta\lambda_1+\lambda_2(\lambda+\lambda_1)}. \tag{4}$$

Equation (4) may be written

$$x^2 - 100A'x = -(100)^2 B'. \quad \therefore x = 50A' - 50\sqrt{(A'^2 - 4B')}.$$

Substituting the numerical values given in prob. 218, I find $x = 20.1293$ and $y = 19.4399$.

VERIFICATION.—In accordance with the conditions, the rate of dividend of Doe's estate is found to be 20.1293 per cent of the liabilities. And the rate of dividend of Roe's estate is found to be 19.4399 per cent of the liabilities. These rates can be verified as follows:

John Doe's estate—direct liabilities,		\$33,425.61
Endorsements for Richard Roe	\$34,949.16	28,155.08
Less dividend from " "	6,794.08	
Total liabilities,		\$61,580.69

Total dividends \$12,395.76, equal to 20.1293 per cent of liabilities.

Richard Roe's estate—direct liabilities,		\$46,212.00
Endorsements for John Doe	\$9,500.00	7,587.72
Less dividend from " "	1,912.28	
Total liabilities,		\$53,799.72

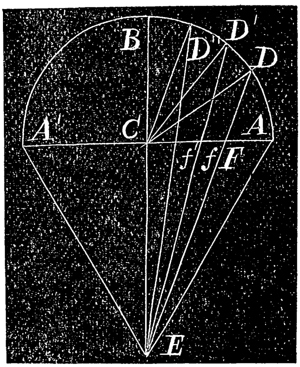
Total dividends, \$10,458.78, equal to 19.4399 per cent of total liabilities.

APPROXIMATE MULTISECTION OF AN ANGLE AND HINTS FOR REDUCING THE UNAVOIDABLE ERROR TO THE SMALLEST AMOUNT.

BY CHAS. H. KUMMELL, DETROIT, MICHIGAN.

THE method of Query, page 96, is applicable also for multisection of angles or for dividing angles in a given ratio, approximately.

For example, if BCD shall be trisected, draw ACA' perpendicular to BC and describe, with any radius CA, the semicircle $ADBA'$; make $AE = A'E = AA'$, join ED intersecting AA' at F; trisect CF at f and f' and draw EfD' and $Ef'D''$, then the arc BD will be approximately trisected in D' and D''.



The answer to the query by Mr. E. B. Seitz at page 125, 126 is quite sufficient to prove this construction to be approximately true; yet for my purpose I shall present a different treatment.

The lines Ef and Ef' will intersect the circle ABA' at points D' and D'' which are more or less distant from the true points required; they may also be on the right or on the left of the true points. Let $CA = CB = 1$; $BCD'' = \varphi$; $BED'' = \psi$; $Cf' = x$. Let φ_0 be the true angle, then